Normal Percentiles

Lecture 21
Section 6.3.1

Robb T. Koether

Hampden-Sydney College

Fri, Oct 3, 2008

Outline

1. Homework Review
2. Hypothesis Testing
3. z-Scores
4. Standard Normal Percentiles
5. Normal percentiles
6. Assignment

Homework Review

Exercise 6.15, page 378.

- A young woman needs a 15-ampere fuse for the electrical system in her apartment and has decided to buy either Brand A or Brand B.
- The length of life for Brand A is approximately normal with a mean of 1000 days and a standard deviation of 30 days.
- The length of life for Brand B is approximately normal with a mean of 990 days and a standard deviation of 10 days.

Homework Review

Exercise 6.15, page 378.

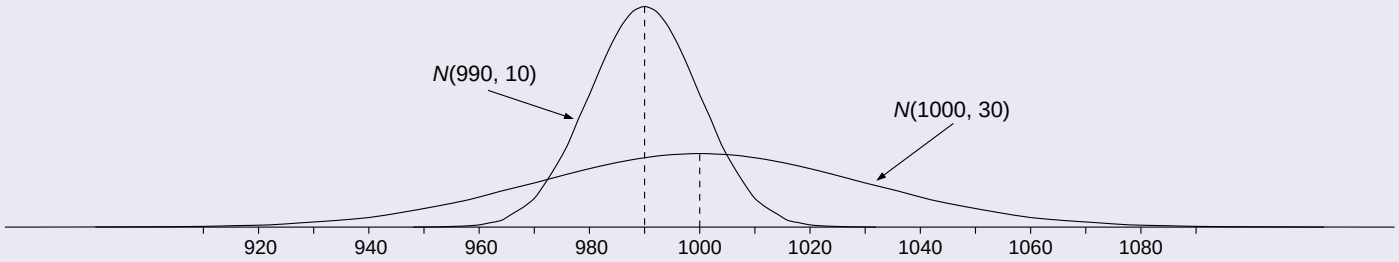
(a) Sketch a picture of the two distributions for the lifetime of fuses. Use the same axis and be sure to label the two distributions appropriately.

(b) The woman would be completely satisfied if the fuse she buys lasts longer than 980 days. Which brand of fuse should she buy? Explain.

Homework Review

Solution

(a)



Homework Review

Solution

(b)

- The probability that the Brand A fuse will last at least 980 hours is

$$\texttt{normalcdf(980,E99,1000,30)} = 0.7475.$$

- The probability that the Brand B fuse will last at least 980 hours is

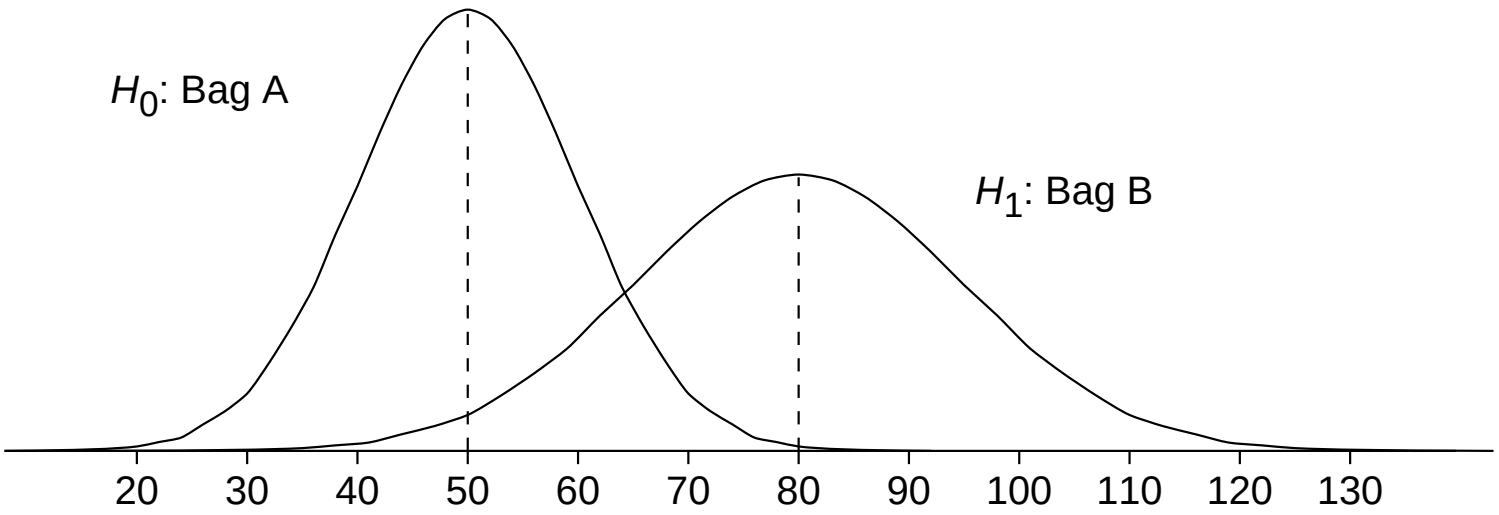
$$\texttt{normalcdf(980,E99,990,10)} = 0.8413.$$

- She should buy Brand B because it is more likely to last at least 980 hours.

Bag A vs. Bag B

- Suppose we have two bags, Bag A and Bag B.
- Each bag contains *millions* of vouchers.
- In Bag A, the values of the vouchers have distribution $N(50, 10)$.
- In Bag B, the values of the vouchers have distribution $N(80, 15)$.

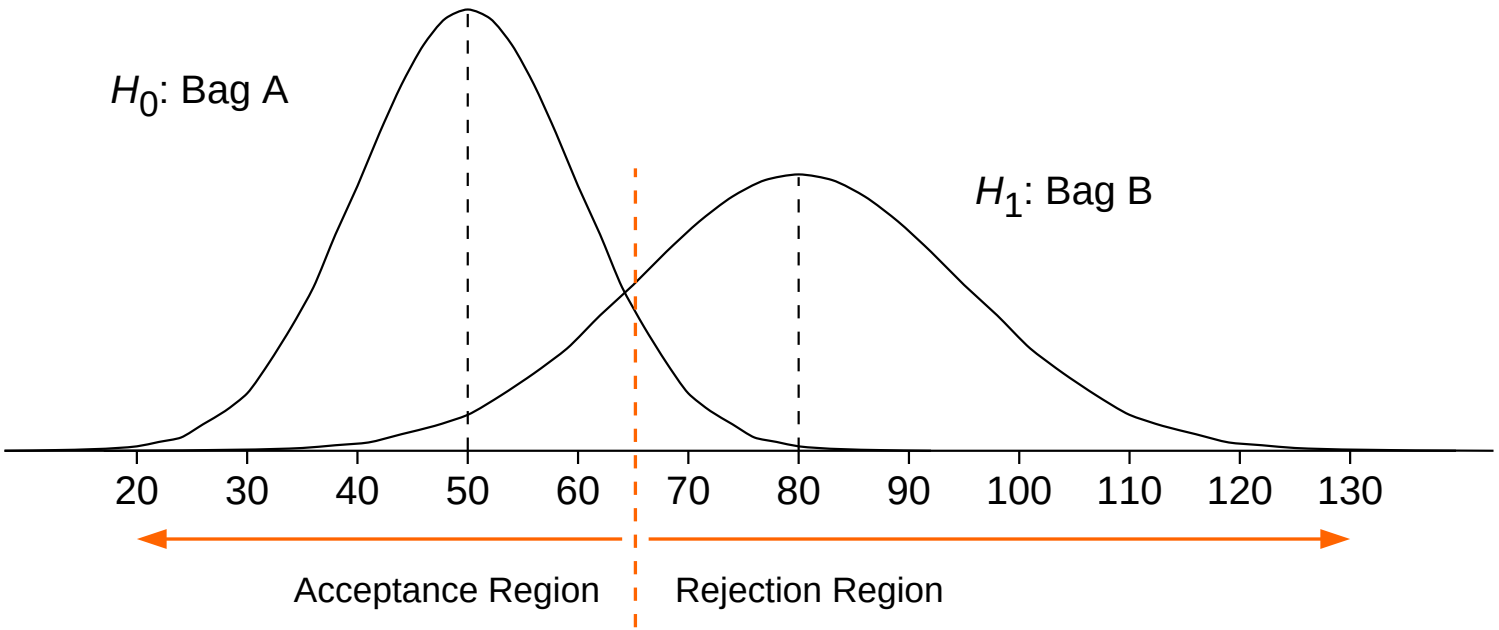
Bag A vs. Bag B



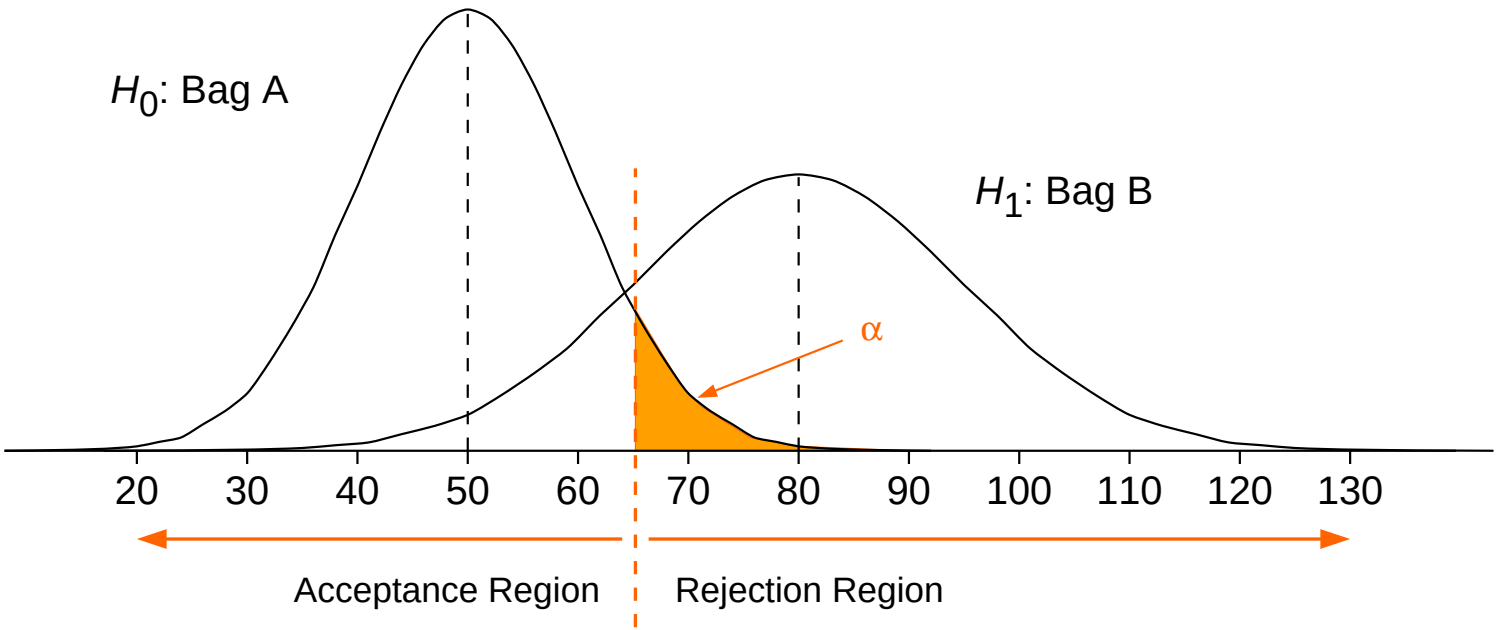
Bag A vs. Bag B

- We select one voucher at random from one bag.
- If its value is less than or equal to \$65, then we will decide that it was from Bag A.
- What is the value of α?
- What is the value of β?

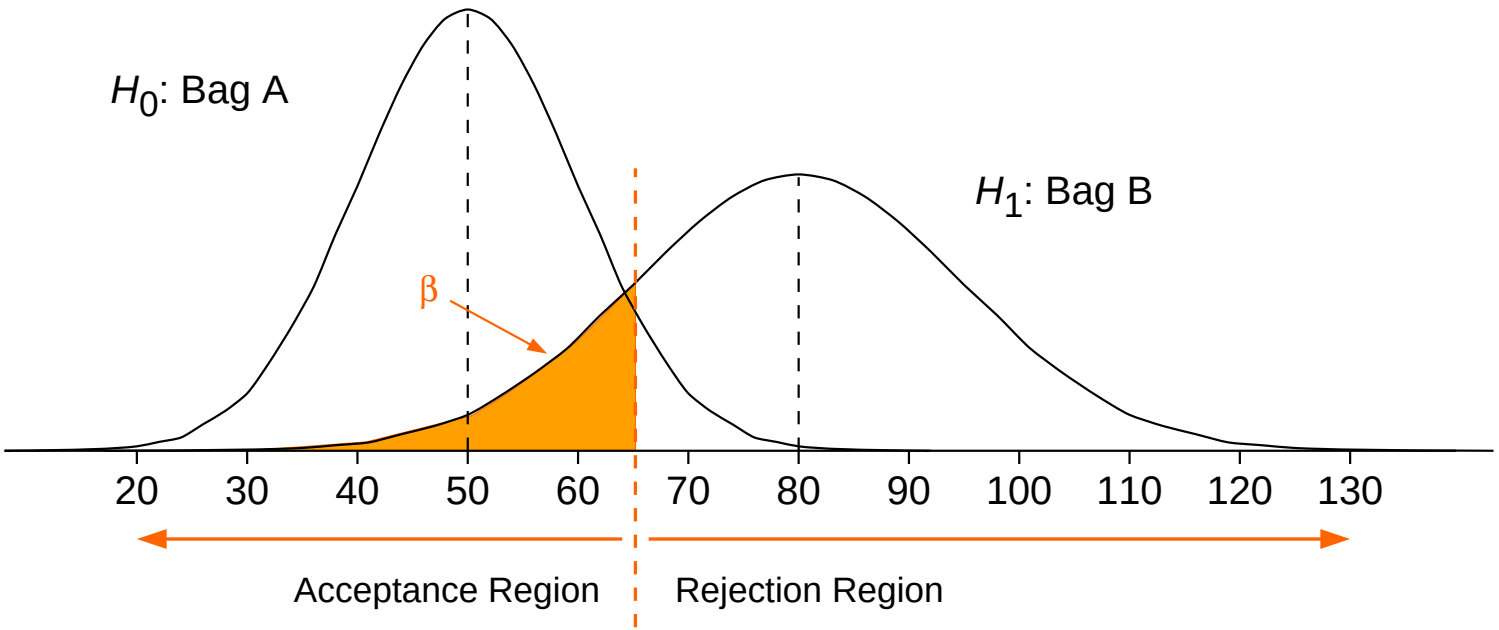
Bag A vs. Bag B


H_0: Bag A
H_1: Bag B
20
30
40
50
60
70
80
90
100
110
120
130
Acceptance Region
Rejection Region

Bag A vs. Bag B


H_0: Bag A
H_1: Bag B
α
20
30
40
50
60
70
80
90
100
110
120
130
Acceptance Region
Rejection Region

Bag A vs. Bag B


H_0: Bag A
H_1: Bag B
β
20
30
40
50
60
70
80
90
100
110
120
130
Acceptance Region
Rejection Region

Bag A vs. Bag B

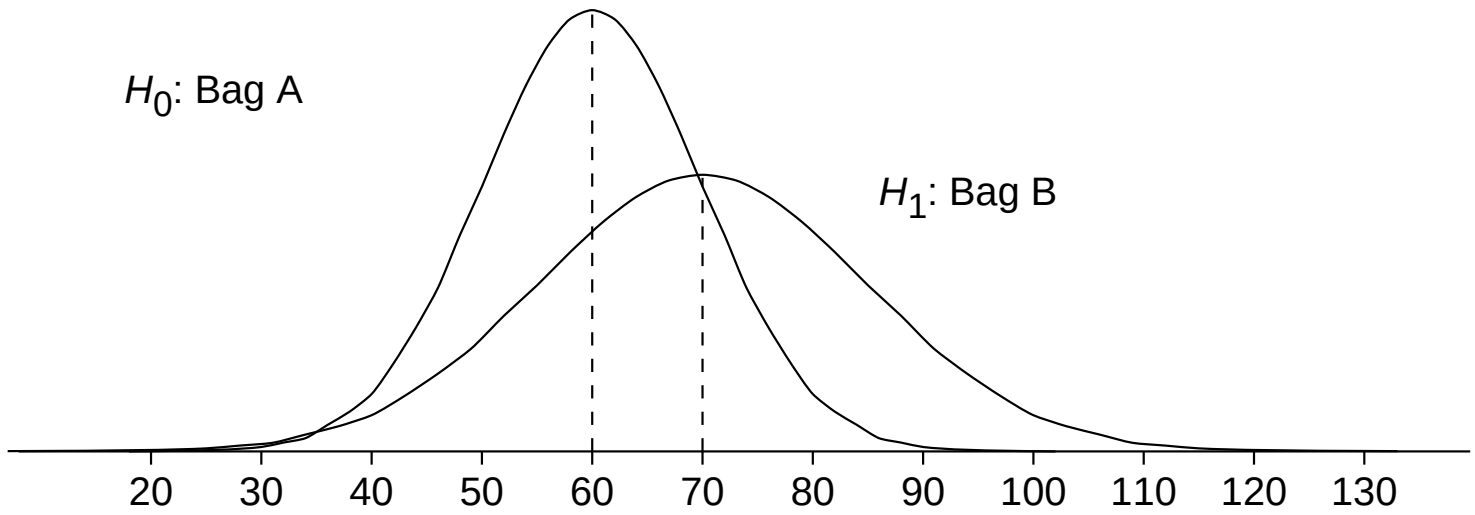
Normal Percentiles
Robb T. Koether
Homework Review
Hypothesis Testing
z-Scores
Standard Normal Percentiles
Normal percentiles
Assignment

- We calculate
  $\alpha = \texttt{normalcdf(65,E99,50,10)} = 0.0668.$
- We calculate
  $\beta = \texttt{normalcdf(-E99,65,80,15)} = 0.1587.$

Bag A vs. Bag B

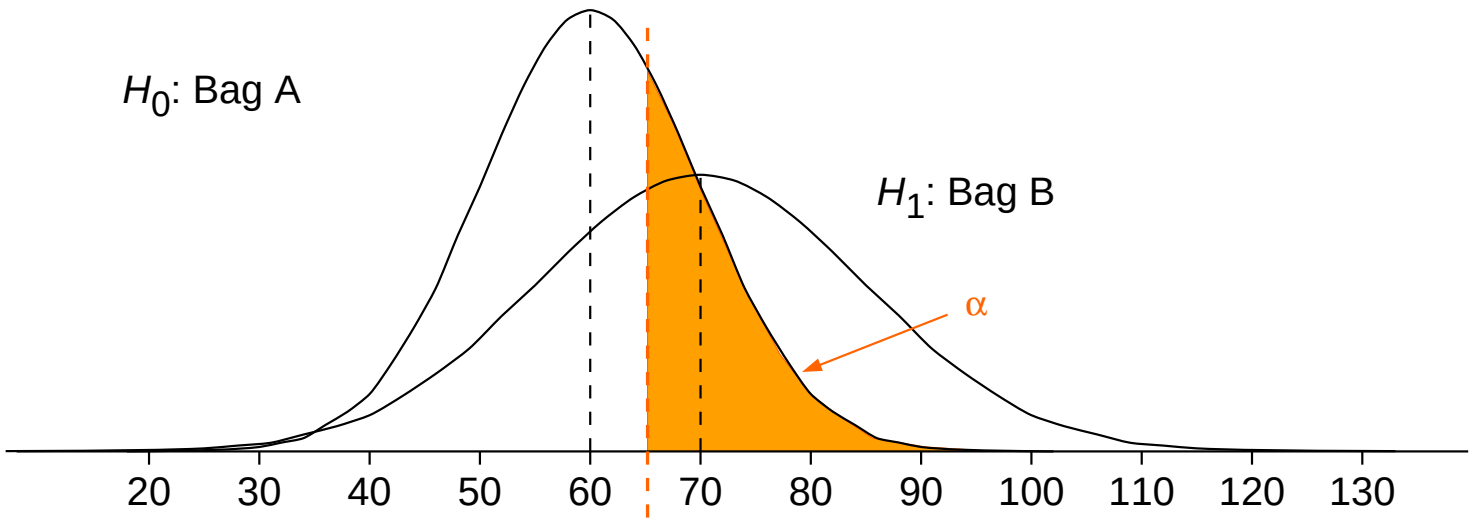
Normal Percentiles
Robb T. Koether
Homework Review
Hypothesis Testing
z-Scores
Standard Normal Percentiles
Normal percentiles
Assignment

- If the distributions are very close together, then α and β will be large.



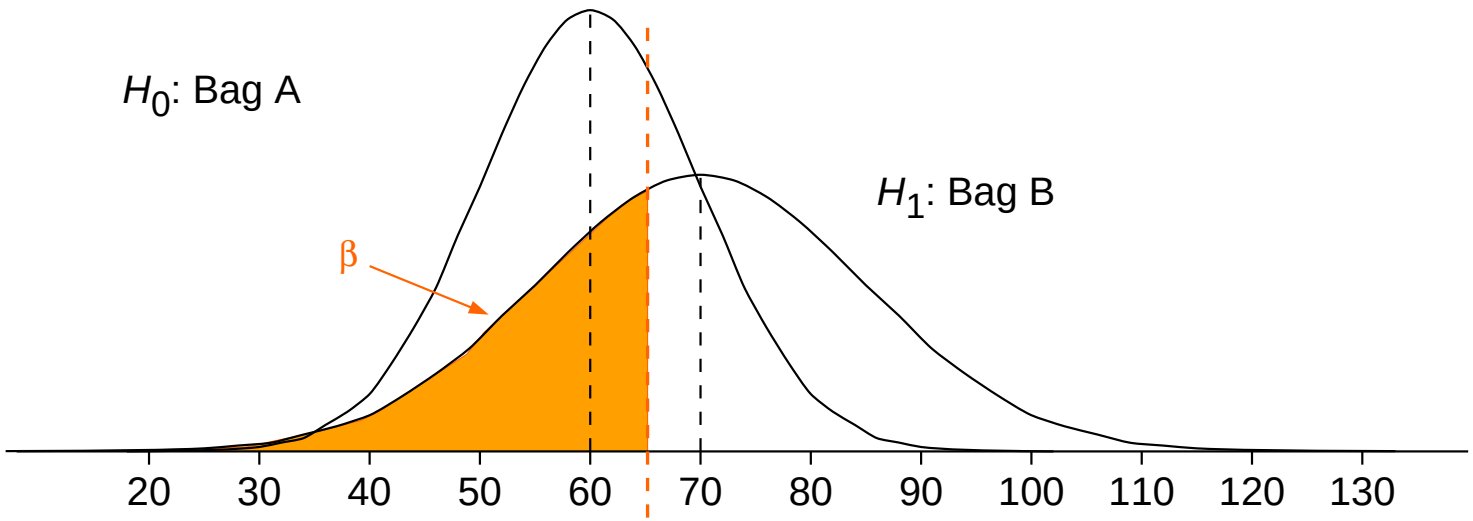
Bag A vs. Bag B

- If the distributions are very similar, then α and β will be large.



Bag A vs. Bag B

- If the distributions are very similar, then α and β will be large.

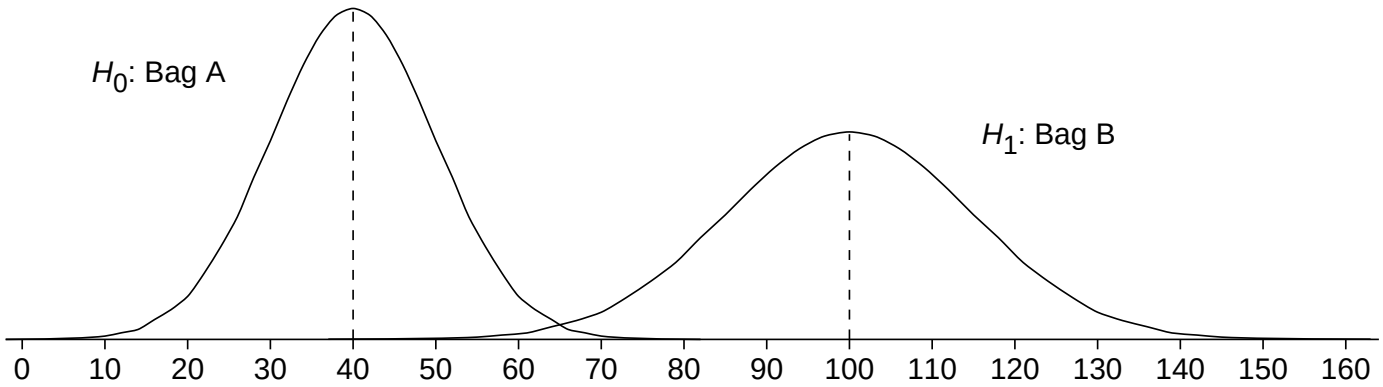


Bag A vs. Bag B

- We calculate
  $\alpha = \texttt{normalcdf(65,E99,60,10)} = 0.3085.$
- We calculate
  $\beta = \texttt{normalcdf(-E99,65,70,15)} = 0.3694.$

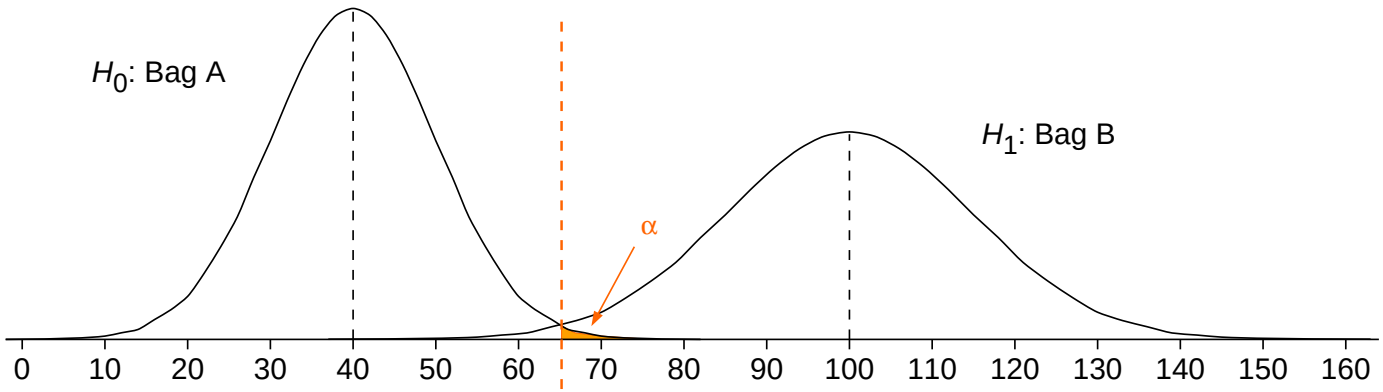
Bag A vs. Bag B

- Similarly, if the distributions are far apart, then α and β will both be very small.



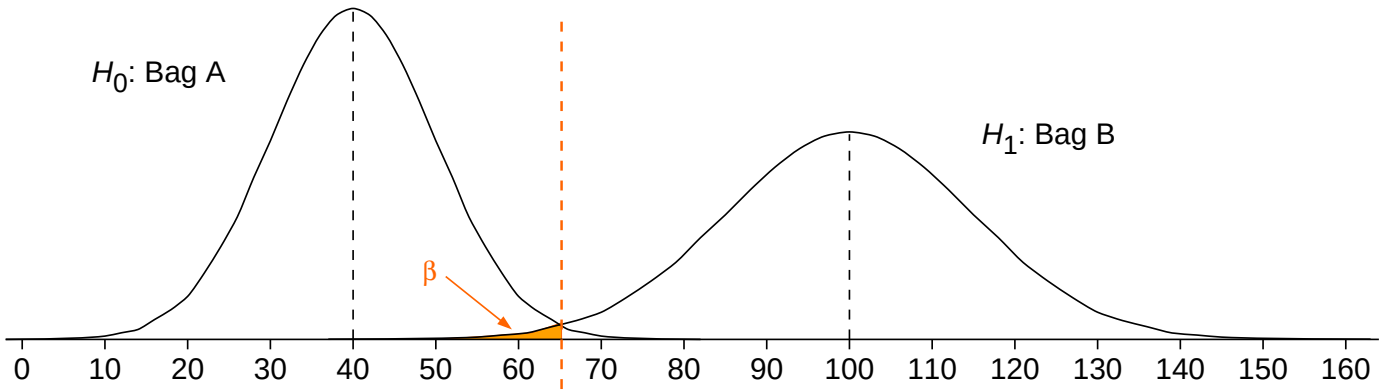
Bag A vs. Bag B

- Similarly, if the distributions are far apart, then α and β will both be very small.



Bag A vs. Bag B

- Similarly, if the distributions are far apart, then α and β will both be very small.



Bag A vs. Bag B

Normal Percentiles
Robb T. Koether
Homework Review
Hypothesis Testing
z-Scores
Standard Normal Percentiles
Normal percentiles
Assignment

- We calculate
  $\alpha = \texttt{normalcdf(65,E99,40,10)} = 0.0062.$
- We calculate
  $\beta = \texttt{normalcdf(-E99,65,100,15)} = 0.0098.$

z-Scores

Definition (z-score (or standard score))

The z-score (or standard score) of a member of a sample or population is the number of standard deviations between that value and the mean.

- Compute the z-score of x as

$$z = \frac{x - \bar{x}}{s}.$$

- Equivalently

$$x = \bar{x} + zs.$$

Areas Under Other Normal Curves

Normal Percentiles
Robb T. Koether
Homework Review
Hypothesis Testing
z-Scores
Standard Normal Percentiles
Normal percentiles
Assignment

- If a variable X has a normal distribution, then the z-scores of X have a standard normal distribution.
- That is, if X is $N(\mu, \sigma)$, then $\dfrac{X - \mu}{\sigma}$ is $N(0, 1)$.
- In other words,

$$Z = \frac{X - \mu}{\sigma}.$$

Example

Example (z-scores)

- Let X be $N(30, 5)$.
- What proportion of values of X are between 25 and 38?
- Compute z-score of $25 = \frac{25-30}{5} = \frac{-5}{5} = -1.0$.
- Compute z-score of $38 = \frac{38-30}{5} = \frac{8}{5} = 1.6$.

Example

Example (z-scores)

- Now find the area between -1.0 and 1.6 under the *standard* normal curve.

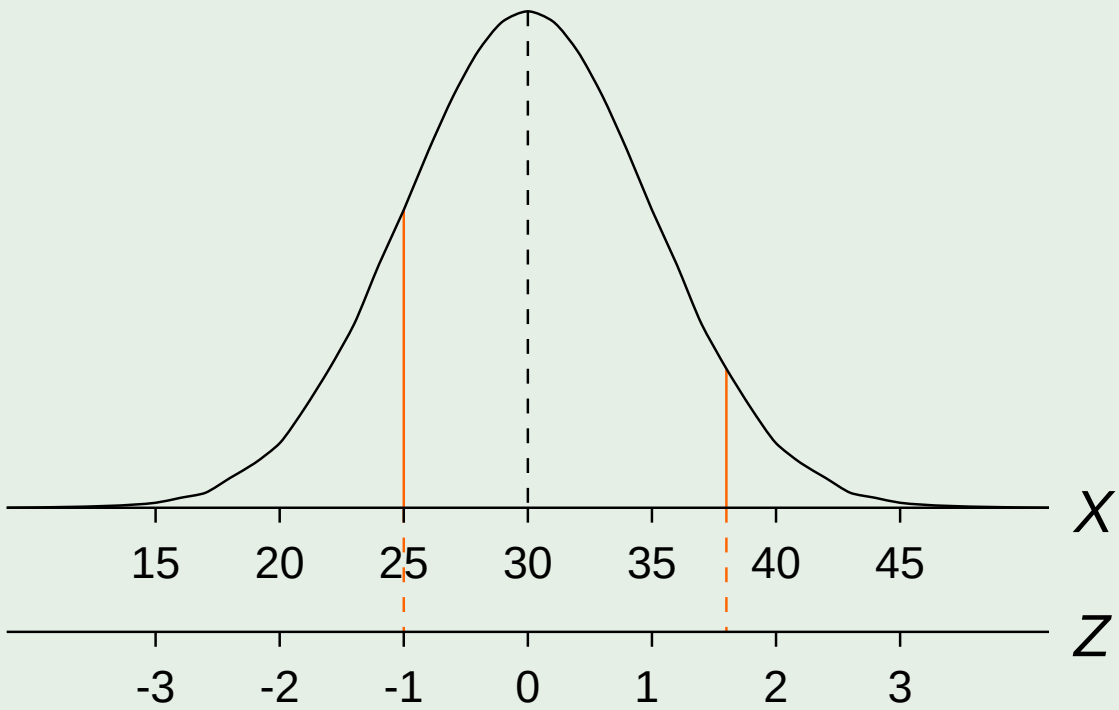
$$\texttt{normalcdf(-1.0,1.6)} = 0.7865.$$

- Therefore, 78.65% of the values of X are between 25 and 38.

Example

Normal Percentiles
Robb T. Koether
Homework Review
Hypothesis Testing
z-Scores
Standard Normal Percentiles
Normal percentiles
Assignment

Example (z-scores)



Two Problems

Normal Percentiles
Robb T. Koether
Homework Review
Hypothesis Testing
z-Scores
Standard Normal Percentiles
Normal percentiles
Assignment

- Find a normal probability:

  Given the value of z $\longrightarrow$ Find the area to the left of z.

- Find a normal percentile:

  Given the area to the left of z $\longrightarrow$ Find the value of z.

- The second problem is the reverse of the first problem.

Standard Normal Percentiles

Example (Standard normal percentile)

- What is the 90th percentile of z?
- That is, find the value of z such that the area to the left is 0.9000.
- On the TI-83, use the `invNorm` function.

TI-83 - Standard Normal Percentiles

TI-83 Standard normal percentiles

- Press `2nd DISTR`.
- Select `invNorm` (Item #3).
- Enter the percentile rank as a decimal (i.e., the area).
- Press `ENTER`. The percentile appears in the display.

TI-83 - Standard Normal Percentiles

Practice

- Use the TI-83 to find the following percentiles.
  - Find the 90^{th} percentile of z.
  - Find the 1^{st} percentile of z.
  - The value of z that cuts off the top 20%.
  - Find Q_1 and Q_3 of z (middle 50%).
  - The values of z that determine the middle 30%.

Normal Percentiles

Normal Percentiles
Robb T. Koether
Homework Review
Hypothesis Testing
z-Scores
Standard Normal Percentiles
Normal percentiles
Assignment

- To find a percentile of a variable X that is $N(\mu, \sigma)$,
  - Find the percentile for z.
  - Use the equation $x = \mu + z\sigma$ to find x.

Example

Example (Normal percentiles)

- Assume that IQ scores are $N(100, 15)$.
- What is the 90^{th} percentile of IQ scores?
- The 90^{th} percentile of z is 1.282.
- Therefore, the 90^{th} percentile for IQ scores is

$$100 + (1.282)(15) = 119.2.$$

- That is, 90% of IQ scores are below 119.2.

TI-83 - Normal Percentiles

Normal Percentiles
Robb T. Koether
Homework Review
Hypothesis Testing
z-Scores
Standard Normal Percentiles
Normal percentiles
Assignment

TI-83 Normal percentiles

- Use `invNorm` to find the standard normal percentile and use the equation $x = \mu + z\sigma$ (as in the previous example).
- Or, use `invNorm` and specify the percentile rank, μ, and σ.

- For example, in the previous problem,

$$\texttt{invNorm(0.90,100,15)} = 119.2.$$

TI-83 - Normal Percentiles

Normal Percentiles
Robb T. Koether
Homework Review
Hypothesis Testing
z-Scores
Standard Normal Percentiles
Normal percentiles
Assignment

Practice

- Find the 80^{th} percentile of IQ scores.
- Find the first and third quartiles of IQ scores.

Assignment

Homework

- Read Section 6.3.1, pages 370 - 376.
- Let's Do It! 6.7, 6.8.
- Exercises 14, 17, 19, 22 - 27, 32, page 376.